Exercise 42

Find the critical numbers of the function.

$$h(t) = 3t - \arcsin t$$

Solution

A critical number is a value of t for which the derivative is zero or nonexistent. Take the derivative of the function.

$$h'(t) = \frac{d}{dt}(3t - \arcsin t)$$

= $3 - \frac{1}{\sqrt{1 - t^2}}$
= $\frac{3\sqrt{1 - t^2}}{\sqrt{1 - t^2}} - \frac{1}{\sqrt{1 - t^2}}$
= $\frac{3\sqrt{1 - t^2} - 1}{\sqrt{1 - t^2}}$

Set what's in the numerator equal to zero and set what's in the denominator equal to zero. Solve these equations for t.

$$3\sqrt{1-t^{2}} - 1 = 0 \qquad \sqrt{1-t^{2}} = 0$$
$$\sqrt{1-t^{2}} = \frac{1}{3} \qquad 1-t^{2} = 0$$
$$1 - t^{2} = \frac{1}{9} \qquad t^{2} = 1$$
$$t^{2} = \frac{8}{9} \qquad t = \pm 1$$
$$t = \pm \frac{\sqrt{8}}{3} \qquad t = \pm 1$$
$$t = \pm 1$$
$$t = -\frac{2\sqrt{2}}{3} \quad \text{or} \quad t = \frac{2\sqrt{2}}{3} \qquad t = -1 \quad \text{or} \quad t = 1$$